

Widths of Ξ Hypernuclear States

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Abstract

The Λ and neutron decay widths of Ξ hypernuclear states, based on calculated $\Xi N \rightarrow \Lambda\Lambda$ mixing amplitudes, are estimated. The widths which result from using the Nijmegen Model D interaction are sufficiently small, of order 1.5 MeV, that experiments to observe Ξ hypernuclear states using the (K^-, K^+) reaction may be feasible.

We would like to dedicate this paper to the memory of Professor Hiroharu Bandō, a warm and courageous man, a friend, and a major contributor to hypernuclear physics.

1 Introduction

The structure properties of hypernuclei reflect the nature of the underlying baryon-baryon interactions, for which there is very little two-body data other than for the nucleon-nucleon interaction, and thus can provide rather stringent tests of models for the free-space YN and YY interactions. Over the years, the Nijmegen group has constructed a number of one-boson-exchange models for the baryon-baryon interaction (NN , $\Lambda N - \Sigma N$, and $\Xi N - \Lambda\Lambda - \Sigma\Sigma - \Lambda\Sigma$) using $SU(3)_{\text{flavor}}$ symmetry to relate baryon-baryon-meson coupling constants and phenomenological hard or soft cores at short distances. More recently, the Jülich group has constructed meson-exchange models for the YN interaction along the lines of the Bonn model for the NN interaction using $SU(6)$ symmetry to relate coupling constants and short-range form factors (reviews of all the relevant models appear in the proceedings of the Shimoda conference on hypernuclear and strange particle physics [1]; in particular, see the reviews by Rijken and Holinde).

To test these models against the considerable body of information on Λ hypernuclei [2, 3, 4], effective interactions appropriate for use in limited spaces of shell-model orbits must be calculated. This has been done by calculating nuclear-matter G -matrices [5, 6] for the coupled ΛN and ΣN channels. For most models, the resulting Λ well depth is in reasonable agreement with the empirical value deduced from fitting the binding energies of Λ single-particle states [7, 8] observed via the (π^+, K^+) reaction [9], but the partial-wave contributions vary widely for the different models. In fact, the spin-dependence of the central interaction varies considerably in the different models [5]. While empirical ΛN effective interactions fitted to the properties of light Λ hypernuclei are more attractive in the singlet than the triplet state [10, 11], a conclusive choice between the models cannot be made since the effective

ΛN interaction resulting from $\Lambda N - \Sigma N$ coupling, which contributes strongly to the spin-dependence for $A=4$ [12], has yet to be evaluated in a consistent way.

The baryon-baryon interactions which result from an extension of the boson-exchange models to the $S = -2$ sector can be qualitatively quite different from one another at long range [13, 14, 15], even before the uncertainties at short range, where quark model calculations may provide some guidance (K. Shimizu in Ref. [1]), are considered. Rather little is known about $S = -2$ systems. There are only a few examples of $\Lambda\Lambda$ hypernuclear events seen in emulsion experiments with K^- beams, and a few others attributed to the formation of Ξ hypernuclei (see Ref. [16] for references). The binding energies of the three examples of light $\Lambda\Lambda$ hypernuclei indicate that the effective $\Lambda\Lambda$ interaction is quite strongly attractive, with a relative $0s$ matrix element of magnitude ~ 4.5 MeV. Also, when taken at face value, the Ξ hypernuclear events indicate an attractive Ξ -nucleus potential with a strength roughly comparable to that for Λ hypernuclei [17]. When the short-range behavior of the YY interaction is chosen to be similar to that of the NN interaction, only Model D of the Nijmegen interactions can provide sufficient attraction to account for the empirical $\Lambda\Lambda$ matrix element [15, 18]. Model D also gives an attractive Ξ -nucleus potential [13]. In contrast, Model F gives repulsion in both cases.

Improved data for the $S = -2$ sector would put strong constraints on baryon-baryon interaction models. To this end, an active experimental program is being pursued at the KEK laboratory in Japan using the double strangeness exchange reaction on nuclear targets [1, 19, 20]. At the Brookhaven AGS, a more intense separated 1–2 GeV/c K^- beam [21] is now available for (K^-, K^+) studies. The first use of this beam line has been in a search for the H dibaryon but new proposals are focussed on more general studies of $S = -2$ spectroscopy. Several of these concentrate on the detection of $\Lambda\Lambda$ hypernuclei produced by Ξ^- interactions with nuclei. Another attractive possibility would be to look for Ξ hypernuclear states using the (K^-, K^+) reaction in analogy to studies of Λ hypernuclei using the (π^+, K^+) reaction. Unfortunately, the (K^-, K^+) cross sections leading to single-particle Ξ configurations are small, of the order of a few hundred nb/sr for light systems [17, 16, 22], and somewhat below current detection capabilities. Nevertheless, an important question, should this type of experiment become feasible, concerns the width of Ξ hypernuclear states which arises from the mixing of Ξ and $\Lambda\Lambda$ hypernuclear states and depends directly on the strength of the $\Xi N \rightarrow \Lambda\Lambda$ interaction. There have been two recent estimates of these widths [16, 22] using rather different approaches. Our own estimates [16] of the decay (escape) widths arising from strong n , p or Λ emission from Ξ -nucleus configurations were made using standard shell-model techniques. For pedagogical reasons related to the simplicity of nuclear parentages, we concentrated on calculations for a ^{15}N target. In this paper, we make estimates for the escape widths of bound Ξ^- hypernuclear states likely to be produced in the more practical $^{12}\text{C}(K^-, K^+)_{\Xi}^{12}\text{Be}$ reaction considered by Ikeda *et al.* [22]. While the two calculations give comparable estimates of the widths for $\Xi N \rightarrow \Lambda\Lambda$ interactions of the same strength, the partial decay widths from different sources are quite different. Consequently, we point out some of the reasons for these differences.

2 Single-particle Ξ States

The early emulsion data which suggested the existence of bound Ξ states in various nuclear systems were analyzed by Dover and Gal [17] in terms of a Ξ -nucleus single-particle Woods-Saxon well with a depth $V_{\Xi} = 24$ MeV, radius parameter $r_0 = 1.1$ fm and diffuseness $a = 0.65$ fm (or a depth of 21 MeV for $r_0 = 1.25$ fm). This value of V_{Ξ} , comparable to that for the Λ -nucleus potential [7, 8], binds a Ξ^- in both $0s$ and $0p$ states to a ^{11}B core with binding energies of 10.7 and 1.5 MeV, respectively [22]. Since stable targets have $N \geq Z$, the Ξ states of interest will be essentially pure Ξ^- charge states (with $T = -T_z$).

The spectrum for the $^{12}\text{C}(K^-, K^+)_{\Xi}^{12}\text{Be}$ reaction should be qualitatively very similar (see Fig. 1 of Ref. [22]) to that for the well-studied $^{12}\text{C}(\pi^+, K^+)_{\Lambda}^{12}\text{C}$ reaction [9, 23, 24]. The cross sections will, however, be considerably smaller on account of the smaller elementary cross section and the somewhat higher momentum transfer (around 420 MeV/c to 500 MeV/c over the angular range $0^\circ - 10^\circ$ compared with 330 – 365 MeV/c over the same range, respectively). The distorted-wave impulse approximation calculations for $p_K = 1.6$ GeV/c by Ikeda *et al.* [22] for the natural parity $j_N^{-1}j_{\Xi}$ configurations give 0° cross sections of $0.25 \mu\text{b/sr}$ for the 1^- ground-state peak and $0.50 \mu\text{b/sr}$ for the $0^+, 2^+$ complex of p_{Ξ} states, if the recent KEK measurement of the elementary cross section (averaged over θ_{lab} from 1.7° to 13.6°) by Iijima *et al.* [20] is used. These cross sections at $p_K = 1.6$ GeV should be multiplied by ~ 0.71 , representing the parentage for proton pickup from ^{12}C to the ground state of ^{11}B . However, this factor is gained back at $p_K = 1.8$ GeV/c where the elementary cross section appears to peak (see Fig. 5 in Ref. [17]) and for which the D6 beamline at Brookhaven is optimized [21].

The relevant basis for comparison for these predicted cross sections is the sensitivity of $\sim 1.0 \mu\text{b/sr}$ achieved for the (π^+, K^+) reaction with a flux of $10^7 \pi^+$ on target per spill and an energy resolution of ~ 3 MeV [9]. For K^- at 1.8 GeV, a flux of 2.8×10^6 per 10^{13} protons with a π^-/K^- of 2.5:1 was typical in 1992 (note added in proof to Ref. [21]); the flux is limited by the proton flux that the production target can stand but runs have been made using 1.5×10^{13} protons per spill. The question of how much improved resolution can improve the sensitivity depends on the natural width of the Ξ hypernuclear states, each of which is broadened by the strong decay processes

$$\Xi^- p \rightarrow \Lambda\Lambda \quad Q = 28.3 \text{ MeV}, \quad (1)$$

$$\Xi^0 n \rightarrow \Lambda\Lambda \quad Q = 23.2 \text{ MeV}. \quad (2)$$

In fact, the first concern is that Ξ hypernuclear states could have widths that are some appreciable fraction of the nuclear matter estimate (~ 10 MeV [17, 16]) and therefore comparable with the spacings between Ξ single-particle levels. Our recent estimates [16] and those of Ikeda *et al.* [22], both of which use $\Xi N \rightarrow \Lambda\Lambda$ conversion potentials based on the Nijmegen Model D interaction, give much smaller widths of around 1.5 MeV. Since the calculated widths depend quadratically on the strength of the conversion potential, which involves the exchange of strange mesons and varies

widely in different baryon-baryon models, the widths could differ considerably from these estimates. Of course, both the positions and widths of Ξ hypernuclear levels, if they exist and can be observed with sufficiently high resolution, together with apparently strong attraction in the $\Lambda\Lambda$ interaction, would put very strong constraints on baryon-baryon interaction models.

In the next section, we present in some detail our calculation of the widths of Ξ hypernuclear states based on the mixing of Ξ and $\Lambda\Lambda$ hypernuclear states through the $\Xi N \rightarrow \Lambda\Lambda$ interaction. We concentrate on the case of states in ${}^{12}_{\Xi}\text{Be}$ since ${}^{12}\text{C}$ is the most likely target for attempts to observe bound Ξ hypernuclear states using the (K^-, K^+) reaction. We also compare our calculation with that of Ikeda *et al.* [22].

3 Mixing of Ξ and $\Lambda\Lambda$ Hypernuclear States

It is the admixtures of $\Lambda\Lambda$ hypernuclear states into a Ξ hypernuclear state which give the state its decay width into single Λ , nucleon and other channels. The advantage of a microscopic shell-model treatment is that the decay energies are defined in terms of the energies of the nuclear cores and the interaction of the two Λ 's with these cores and each other. The decay energies are then quite well determined in terms of our empirical knowledge of single and double Λ hypernuclei. In practice, we make a weak-coupling approximation in which we couple the hyperons to specific nuclear core states and do not consider the mixing of these states in a shell-model diagonalization; there are, however, constraints imposed the fact that there should be no spurious center of mass motion, which we take into account as necessary. In this basis, an estimate of the mixing matrix elements in terms of the two-body matrix elements of the $\Xi N \rightarrow \Lambda\Lambda$ transition potential is straightforward. The actual admixtures are calculated in first-order perturbation theory and the spectroscopic amplitudes for each decay channel are then calculated by standard parentage and recoupling techniques.

3.1 Energies and Decay Thresholds

The energies of the most important Ξ^- and $\Lambda\Lambda$ weak-coupling basis states and the relevant decay channels are given in Fig. 1 for the specific case of a ${}^{12}\text{C}$ target, with the energies measured with respect to the ${}^{12}_{\Lambda\Lambda}\text{Be}$ ground state which is bound with respect to the $\Lambda + \Lambda$ threshold by $B_{\Lambda\Lambda}$ where

$$B_{\Lambda\Lambda} = 2B_{\Lambda}({}^{11}_{\Lambda}\text{Be}) + \Delta B_{\Lambda\Lambda} . \quad (3)$$

The single- Λ binding energies are known quite accurately for many p -shell hypernuclei [2] and can be quite reliably estimated in cases for which a measurement does not exist. For example, in the present case, one obtains $B_{\Lambda}({}^{11}_{\Lambda}\text{Be}) = 10.46$ MeV by adding the 0.22 MeV charge-dependent difference between $B_{\Lambda}({}^{10}_{\Lambda}\text{Be})$ and $B_{\Lambda}({}^{10}_{\Lambda}\text{B})$ to $B_{\Lambda}({}^{11}_{\Lambda}\text{B})$. Alternatively, charge-independent estimates using empirically determined Λ -nucleus potentials [7, 8] would be quite accurate enough for the present purposes. $\Delta B_{\Lambda\Lambda}$ is the interaction energy of a pair of Λ particles in the 1S_0 state and is around

4.5 MeV for the three known examples of $\Lambda\Lambda$ hypernuclei [14, 25]. The $^{11}\text{B} + \Xi^-$ threshold is a further 28.34 - $S_p(^{11}\text{B}) = 17.1$ MeV above the $^{10}\text{Be} + \Lambda + \Lambda$ threshold.

As noted in the previous section, the binding energy of the $0s$ ($0p$) Ξ^- state is about 10.7 (1.5) MeV for the Ξ^- -nucleus potential of Dover and Gal [17], which has a Woods-Saxon form with a well depth of 24 MeV for $r_0 = 1.1$ fm and $a = 0.65$ fm. In the weak-coupling limit, the (K^-, K^+) reaction will populate the Ξ^- states in proportion to the proton pickup spectroscopic factor relating the target and core states [26]. For no spin-flip, the $0s$ states will have spin-parity 1^- and two $0p$ 2^+ states will be most strongly produced, with equal cross sections in the weak-coupling limit. In practice, the 2^+ states will be mixed and separated by about an MeV [26], depending on the ΞN interaction strength and the Ξ spin-orbit splitting [13], which is of the same order as the expected widths of the states.

The $\Lambda\Lambda$ admixtures into the Ξ state, by conversion on a $0s$ - or $0p$ -shell proton, give rise to decay amplitudes into the single-baryon escape channels shown at the left of Fig. 1. The lowest threshold is for neutron emission, as will generally be the case since the transformation of two protons from the p -shell target nucleus leaves a neutron-rich $A-2$ core for the $\Lambda\Lambda$ hypernucleus. For reference, the separation energies with respect to the $\Lambda\Lambda$ hypernuclear ground state are (assuming a constant $\Delta B_{\Lambda\Lambda}$ for all species)

$$S_\Lambda = B_\Lambda(^{11}_\Lambda\text{Be}) + \Delta B_{\Lambda\Lambda} \quad (4)$$

$$S_n = 2B_\Lambda(^{11}_\Lambda\text{Be}) - 2B_\Lambda(^{10}_\Lambda\text{Be}) + S_n(^{10}\text{Be}) \quad (5)$$

$$S_p = 2B_\Lambda(^{11}_\Lambda\text{Be}) - 2B_\Lambda(^{10}_\Lambda\text{Li}) + S_p(^{10}\text{Be}) \quad (6)$$

$$S_\alpha = 2B_\Lambda(^{11}_\Lambda\text{Be}) - 2B_\Lambda(^7_\Lambda\text{He}) + S_\alpha(^{10}\text{Be}) \quad (7)$$

and so on. Also important for neutron emission is the channel in which $^{11}_{\Lambda\Lambda}\text{Be}$ is left with one of the Λ s in a $0s$ state and the other in a $0p$ state. The separation between the $0s$ and $0p$ Λ single-particle states is very close to 10 MeV, a value that we take for definiteness. Of course, the nuclear core may be left in an excited state and this energy is easily taken into account in the weak-coupling approximation.

The $\Lambda\Lambda$ states which are most important for $\Xi^- p \rightarrow \Lambda\Lambda$ conversion from the $0s$ Ξ^- hypernuclear ground state are shown at the right in Fig. 1. In the case of $0s0p \rightarrow 0s0p$ $\Xi^- p \rightarrow \Lambda\Lambda$ conversion, the $0p$ proton parentage of the ^{11}B ground state is spread over four ^{10}Be levels, namely the 0^+ ground state and the first three p -shell 2^+ levels (identified with the 3.37 and 5.96 MeV levels and a level around 10 MeV excitation energy; the 7.54 MeV level and possibly the 9.4 MeV level are predominantly $p^4(sd)^2$ in nature). Also, to a first approximation, we need to take into account conversion only in spin-singlet states since this forces the interaction to take place in a relative $0s$ state for the interacting baryons (the relative p state interaction associated with spin-triplet states will be much weaker). Thus, $\Delta B_{\Lambda\Lambda}$ is the same for the s_Λ^2 and $s_\Lambda p_\Lambda$ singlet configurations and the $^{10}\text{Be}(0_1^+) \otimes ^1P_1(s_\Lambda p_\Lambda)$ has an excitation energy of 10 MeV, determined by the $0s - 0p$ single-particle energy difference. The energy denominators, ΔE_p in Fig. 1, for a perturbative estimate of the admixtures then range from ~ 22 to ~ 12 MeV.

The $\Xi^- p \rightarrow \Lambda\Lambda$ conversion on a $0s$ -shell proton is not so clearcut to describe

and make simple estimates for since the $0s$ nucleon-hole strength occurs at a rather high excitation energy in the core nucleus. The $0s$ proton-hole strength appears as a broad distribution of strength, with a width of 10 MeV or more, in $(p, 2p)$ and $(e, e'p)$ reactions [27]. The corresponding $0s$ neutron strength is seen in the production of Λ hypernuclei at low momentum transfer in the (K^-, π^-) reaction [28]. The spreading of the $0s$ hole strength is apparent even in $1\hbar\omega$ shell-model calculations for p -shell nuclei [29]. The basis for these calculations consists of s^3p^{n-1} and $s^4p^{n-3}(sd)$ configurations (with $n = 8$ for ^{11}B) which are necessarily admixed to ensure that the center of mass is in a $0s$ oscillator state. The s^3p^{n-1} strength then gets spread in the $1\hbar\omega$ shell-model diagonalization, with the major fragments typically spread over 10 MeV or more in excitation energy [29]. Each fragment acquires substantial nucleon escape widths through the $s^4p^{n-3}(sd)$ components in the wave function. The centroid of the observed strength is quite well described in these calculations and the $0s$ proton separation energy in p -shell nuclei can be adequately parametrized by the relation (in MeV) $S_{0s} \sim 20 + 1.88m$, where m is the number of p -shell nucleons, giving $S_{0s} = 33.2$ MeV for ^{11}B .

The corresponding excitation energy in the ^{10}Be core nucleus for the $\Lambda\Lambda$ hypernuclear states is $S_{0s} - S_p(^{11}\text{B}) = 22$ MeV. Then, the energy separation ΔE_s (see Fig. 1) between the $^{12}_{\Xi}\text{Be}$ $0s$ state and the centroid of the s^2_{Λ} configurations based on the $0s$ proton hole strength in ^{11}B is given (in MeV) by

$$\Delta E_s = 28.34 + B_{\Lambda\Lambda} - S_{0s} - B_{\Xi^-}, \quad (8)$$

leading to a rough estimate for ΔE_s of 10 MeV for $A=12$. Our width estimates are obtained by concentrating the $0s$ hole strength at its centroid energy and making a perturbative calculation of the admixture of the $\Lambda\Lambda$ state based on this pure $0s$ -hole state.

3.2 Mixing and Decay Amplitudes

The wave function of a strangeness -2 hypernuclear state containing the lowest-energy Ξ configuration with admixtures of the configurations considered above will be of the form (the basis is formed from $1\hbar\omega$ excitation energy configurations with respect to the lowest $\Lambda\Lambda$ configurations)

$$a|0s^40p^{n-1}0s_{\Xi}\rangle + b|0s^40p^{n-2}0s_{\Lambda}0p_{\Lambda}\rangle + c|(\alpha 0s^30p^{n-1} + \beta 0s^40p^{n-3}(1s0d))0s_{\Lambda}^2\rangle. \quad (9)$$

The coefficient β provides immediate access to s -wave and d -wave neutron emission channels. In our weak-coupling approximation to the full shell-model calculation, we take the pure $^{11}\text{B}(g.s.) \otimes s_{\Xi}$ state as the Ξ^- hypernuclear ground state and make perturbative estimates of the amplitudes b and c . Some constraints are imposed on the amplitude coefficients in Eq. 9, particularly α and β , by the condition that the center of mass of the system remain unexcited (in practice, in a $0s$ harmonic oscillator state). It should also be noted that the condition that the $1\hbar\omega$ $\Lambda\Lambda$ hypernuclear states be non-spurious implies a relationship between the amplitude b and certain of the amplitudes c in Eq. 9.

For a target specified by A (N and Z), $(A-2)\otimes s_\Lambda p_\Lambda$ configurations are admixed when the $0s \Xi^-$ converts on a $0p$ proton, with an amplitude proportional to a single two-body matrix element connecting $0p_N 0s_\Xi$ and $0s_\Lambda 0p_\Lambda$ states. Which $0\hbar\omega$ A-2 core states must be considered depends on the parentage distribution of the $0p$ proton pickup strength from the A-1 core of the Ξ^- hypernuclear state.

The $(A-2)s_N^{-1} \otimes s_\Lambda^2$ configurations are admixed when the $0s \Xi^-$ converts on a $0s$ proton, with an amplitude proportional to a two-body matrix element connecting $0s_N 0s_\Xi$ and $0s_\Lambda^2$ states. We consider the single A-2 core state which corresponds to a pure $0s$ hole state with respect to the $(A-1)g.s.$ (in reality, this $0s$ -hole strength is fragmented over a considerable range of excitation energy). In effect, one orthogonalizes $0s^3 0p^{n-1}$ states, with $0p^{n-1}$ parts identical to the $(A-1)g.s.$ wave function, to spurious $1\hbar\omega$ center of mass excitations of the A-2 nucleus. In general, there will be two possible J and T values for the $0s$ hole states. The fractional reduction in $0s$ proton occupancy, with respect to the naive shell-model value of 2, is represented by

$$\bar{\alpha}^2 = 1 - \frac{(Z-3)}{2(A-2)}, \quad \bar{\beta}^2 = \frac{(Z-3)}{2(A-2)}. \quad (10)$$

The quantities $\bar{\alpha}$ and $\bar{\beta}$ are closely related to, but not exactly equal to, α and β in Eq. 9 since they represent averages over several J and T values and, in addition, the two $s^3 p^{n-1}$ configurations in Eq. 9 are not identical as a result of the orthogonalization process.

To calculate the two-body matrix elements required for the mixing calculation, we take the Gaussian conversion potential for $p\Xi^- \rightarrow \Lambda\Lambda$ used by Myint in Ref. [1] and ignore, for simplicity, the mass differences between the interacting baryons. Then, any two-body matrix elements can be simply expressed in terms of the Talmi integrals

$$I_p = \frac{V_0}{(1 + 2b^2/\mu^2)^{p+3/2}}, \quad (11)$$

where, for Myint's potential, $V_0 = 57.872$ MeV and $\mu = 0.855$ fm. For $b = 1.64$ fm, appropriate to ^{12}C , $I_0 = 2.39$ MeV. For comparison, we deduce $I_0 = 2.44$ MeV from the $0s^2$ matrix element quoted by Ikeda *et al.* [22], who use a δ -function interaction, also based on the Nijmegen Model D potential. The diagonal $0s^2$ and $0s0p$ LS -coupling matrix elements, for the mixing calculations discussed above, take the values I_0 and $\sqrt{1/2}I_0$, respectively, where the factor $\sqrt{1/2}$ arises from the relative $0s$ content of the $p\Xi^-$ state. In the latter case, the projections of $0p_{3/2}0s_{1/2}$ and $0p_{1/2}0s_{1/2}$ configurations on $L=1, S=0$ are $\sqrt{2/3}$ and $\sqrt{1/3}$, respectively.

The conversion from $0p \Xi^-$ hypernuclear states can be treated within the same framework with the primary admixtures coming from $2\hbar\omega$ $\Lambda\Lambda$ hypernuclear states although $0\hbar\omega$ admixtures are possible with smaller matrix elements and larger energy denominators (in a perturbative calculation).

Having calculated spectroscopic factors S_B for baryon emission channels, we estimate partial decay widths for each exit channel using the relations

$$\Gamma = S_B \Gamma_{sp}, \quad \Gamma_{sp} = 2f\gamma P_l. \quad (12)$$

For $f = 1$, Γ_{sp} is a single-particle width evaluated according to Appendix 3F-2 of Bohr and Mottelson's book [30]. The penetrability P_l (denoted by s_l in Ref. [30]) and γ (defined by Eq. (3F-42) of Ref. [30]) are evaluated for a square well potential of radius R and depth V_0 . For s waves,

$$\Gamma_{sp} = 2f \frac{\hbar^2}{MR^2} kR, \quad (13)$$

where k is the wave number of relative motion. For the rather high decay energies relevant to the decay of Ξ hypernuclear states, Γ_{sp} for p waves is close to this value and Γ_{sp} for d waves is an appreciable fraction of it. Michaud *et al.* [31] have shown that the reflection properties of a diffuse well can be approximated by using an equivalent square-well potential and a reflection factor f , which typically has a value around 2.5 for nucleons. The single-particle widths obtained using this prescription compare quite well with those obtained by solving the Schrödinger equation for the complex energies of resonances in a realistic Woods-Saxon well (this can only be done for energies smaller than the widths), although there is an indication that f could be somewhat larger than 2.5 for the rather diffuse light systems of interest.

To take a purely nuclear example of the calculation of decay widths, we consider the $0s$ proton-hole state in the ^{16}O closed shell. The 20% $0s^4 0p^{10}(1s0d)$ admixture gives rise to nucleon decay amplitudes to $0\hbar\omega$ states of the $A=14$ system with excitation energies up to about 11 MeV. Since the $0s$ strength is centered at ~ 32 MeV excitation energy in ^{15}N , the decay energies range from 22 to 11 MeV. Using spectroscopic factors calculated from the pure $0s$ hole state to p -shell states of ^{14}N and ^{14}C and estimates of full single-particle widths for a diffuse potential well at the appropriate decay energies (using Eq. 12), we calculate an escape width for a pure $0s$ -hole state located at the centroid energy of about 8.5 MeV ($\Gamma_n = 5.4$ MeV). To compare with the experimentally observed distribution of $0s$ -hole strength, we would have to fold escape widths with the distribution of hole strength obtained in a shell-model calculation (spreading width) and with the response function for the excitation process (for example, an $(e, e'p)$ reaction). Since the crude estimate for the escape width is already less than a factor of two of the observed width, the width obtained by taking into account the spreading width should be in considerably better agreement with the data, which gives us confidence that our procedure will give reasonable results in hypernuclear applications.

3.2.1 $0s_\Lambda 0p_\Lambda$ admixtures into the $0s \Xi^-$ state

We write the wave function of the $1^-, 0s \Xi^-$ state with $0s_\Lambda 0p_\Lambda$ admixtures as (cf. Eq. 9)

$$|0s \Xi^-\rangle = |^{11}\text{B} \otimes s_\Xi\rangle + \sum_n b(\alpha_n J_n) |^{10}\text{Be}(J_n^\pi) \otimes s_\Lambda p_\Lambda\rangle, \quad (14)$$

where

$$b(\alpha_n J_n) = M(\alpha_n J_n) / \Delta E(\alpha_n J_n) \quad (15)$$

is the perturbative admixture of each $\Lambda\Lambda$ state and $M(\alpha_n J_n)$ is the corresponding mixing matrix element given by

$$M(\alpha_n J_n) = C \sum_j \theta_j U(J_n j J \frac{1}{2}, \frac{3}{2} 1) \times \langle (p\Xi^-) 0p_j 0s_{1/2} | V(p\Xi^- \rightarrow \Lambda\Lambda) | (\Lambda\Lambda)(0s0p) L=1 S=0 T=0 \rangle \quad (16)$$

where θ_j is the spectroscopic amplitude for removal of a $0p$ nucleon with angular momentum j from the ^{11}B ground state [32], C is an isospin Clebsch-Gordan coefficient taking the value $-\sqrt{2/3}$ (for the ground state and first three p -shell 2^+ states of ^{10}Be , $\sum_{nj} C^2 \theta_j^2 = 2.71$ out of 3 p -shell protons) and U is a unitary recoupling coefficient, with arguments $(J_{\text{Be}j_p} J_{j_\Xi}, J_{\text{B}} J_{\Lambda\Lambda})$, which ensures that j_p and j_Ξ are coupled to $J_{\Lambda\Lambda}$. The two-body matrix element is equal to $\sqrt{(2j+1)/12} I_0$. Both the $0p_{3/2}$ and $0p_{1/2}$ amplitudes are important and interfere constructively for 2_2^+ . The mixing amplitudes $b(\alpha_n J_n)$ and the energy denominators ΔE_p are given in Table 1. The total intensity of $s_\Lambda p_\Lambda$ admixtures associated with the four ^{10}Be core states considered is $\sim 0.88\%$.

The spectroscopic amplitudes for p -wave Λ emission to final states of the form $^{10}\text{Be}(J_n^\pi) \otimes 0s_\Lambda$ are obtained by recoupling the wave function. After a sum over the final-state spins is performed, the spectroscopic factors are simply equal to $b^2(J_n^\pi)$. Table 1 gives the decay energy E_Λ for each core state, the single-particle Λ width for this decay energy, and the partial width Γ_Λ for each core state. The total width for p -wave Λ emission sums to 398 keV.

The spectroscopic amplitudes for p -wave neutron emission to final states of the form $^9\text{Be}(J_c) \otimes s_\Lambda p_\Lambda$ are obtained by removing a neutron from the ^{10}Be core state and recoupling the wave function. After a sum over the final-state spins is performed, the recoupling coefficients disappear and we are left with

$$S(0p) = \sum_{\alpha_n \alpha'_n J_n j_p} b(\alpha_n J_n) b(\alpha'_n J_n) \langle \alpha_n J_n || a_{j_p}^\dagger || J_c \rangle \langle \alpha'_n J_n || a_{j_p}^\dagger || J_c \rangle \quad (17)$$

Table 2 lists the ^9Be core states of importance, their excitation energies, the decay energy E_n for each core state, $S(0p)$, the single-particle neutron width for the decay energy, and the partial width Γ_n for each core state. The total width for p -wave neutron emission sums to 622 keV. The effect of coherence amongst the admixed amplitudes for different ^{10}Be 2^+ states in Eq. 17 is significant. The summed spectroscopic strength for neutron removal from each of the first three ^{10}Be states to the ^9Be states listed in Table 2 is in the range 2.5 – 3.0 (out of four p -shell neutrons), while the fourth ^{10}Be state contributes little (these results follow from the supermultiplet structure of the p -shell wave functions).

3.2.2 $0s_\Lambda^2$ admixtures into the $0s \Xi^-$ state

If we took an $s^3 p^7$ shell-model configuration, with $J^\pi; T=1^-; 1$ and the p^7 configuration being the p -shell $3/2^-$ ground state for ^{11}B , to represent the $0s$ proton hole state in ^{11}B , the mixing matrix element in question would be

$$\langle s^4 p^{n-1} s_\Xi | V(p\Xi^- \rightarrow \Lambda\Lambda) | s^3 p^{n-1} s_\Lambda^2 \rangle = \langle s^4 s_\Xi | V | s^3 s_\Lambda^2 \rangle$$

$$= \sqrt{1/2} \langle (p\Xi^-)(0s0s) | V | (\Lambda\Lambda)(0s0s) L=0 \ S=0 \ T=0 \rangle \quad (18)$$

since the p -shell configurations factor out of the matrix element. The matrix element in Eq. 18 must be reduced by a center of mass correction factor which is essentially, but not quite, the α in Eq. 9. To make this point explicit, and to make an analytic but realistic calculation, we take the ^{11}B ground state wave function to be the $3/2^-$ state with [443] spatial symmetry and $K=3/2$; the SU(3) symmetry is (1 3). Namely,

$$|^{11}\text{B}_{g.s.}\rangle = \sqrt{\frac{21}{26}} |(1\ 3)L=1\ S=\frac{1}{2}\rangle - \sqrt{\frac{5}{26}} |(1\ 3)L=2\ S=\frac{1}{2}\rangle. \quad (19)$$

This configuration accounts for somewhat over 80% of the p -shell wave function with a typical effective interaction. Then, the s^3p^7 0s-hole state with $J^\pi=1^-$ and $T=1$ is

$$|s^3 \otimes ^{11}\text{B}_{g.s.}\rangle = \sqrt{\frac{14}{26}} |(1\ 3)L=1\ S=0\rangle - \sqrt{\frac{7}{26}} |(1\ 3)L=1\ S=1\rangle + \sqrt{\frac{5}{26}} |(1\ 3)L=2\ S=1\rangle. \quad (20)$$

The structure of the spurious center of mass states to which the state in Eq. 20 must be orthogonalized are independent of L and can be written

$$\begin{aligned} |(1\ 3)T=1\ S=0\rangle &= -\sqrt{\frac{3}{10}} |s^3p^7\rangle + \sqrt{\frac{8}{15}} |(3\ 1)\frac{1}{2}\frac{1}{2}\rangle - \sqrt{\frac{1}{30}} |(1\ 2)\frac{1}{2}\frac{1}{2}\rangle + \sqrt{\frac{2}{15}} |(1\ 2)\frac{3}{2}\frac{1}{2}\rangle \\ |(1\ 3)T=1\ S=1\rangle &= \sqrt{\frac{1}{10}} |s^3p^7\rangle + \sqrt{\frac{1}{10}} |(1\ 2)\frac{1}{2}\frac{1}{2}\rangle + \sqrt{\frac{4}{10}} |(1\ 2)\frac{1}{2}\frac{3}{2}\rangle + \sqrt{\frac{4}{10}} |(1\ 2)\frac{3}{2}\frac{1}{2}\rangle, \end{aligned} \quad (21)$$

where we have labelled the $s^4p^5(sd)$ configurations by giving only $(\lambda\ \mu)TS$ for the p^5 configuration (the configurations with $S=0$ and $S=1$ have spatial symmetries [442] and [433] respectively). The overlaps of the s^3p^7 state in Eq. 20 with the spurious states of the appropriate L can be read off from Eqs. 21 and the non-spurious 0s-hole state obtained by Schmidt orthogonalization. The $J^\pi = 2^-$ 0s-hole state can be similarly constructed and the total 0s occupancy of 1.7 protons in the wave function of Eq. 19 is shared between the 1^- and 2^- states in the ratio $\sqrt{3/5}.\sqrt{103/115}$.

The overlap between the wave function of Eq. 20 and the corresponding non-spurious state is $\sqrt{103/130}$ and this multiplied by $\sqrt{1/2}I_0$ (the two-body matrix element in Eq. 18 is equal to I_0) gives the required mixing matrix element; the mixing matrix element for $I_0 = 2.39$ MeV is 1.507 MeV. Taking ΔE_s equal to 9.8 MeV from Fig. 1, we obtain a perturbative estimate of a 2.4% admixture of the $^{10}\text{Be}(s_N^{-1}) \otimes s_\Lambda^2$ configuration ($c^2 = 0.024$ in Eq. 9).

The s_Λ^2 admixtures in the 0s Ξ hypernuclear state provide a mechanism for nucleon decay to $s^4p^5 \otimes s_\Lambda^2$ hypernuclear states, in particular for neutron emission which has the lowest threshold and largest decay energy of 22.3 MeV (9.9 MeV for proton emission). To estimate, for example, the spectroscopic factors for neutron decay to $\Lambda\Lambda$ hypernuclear states with the ^9Be core states listed in Table 2, one looks for the $s^4p^5(sd)$ component in the non-spurious 1^- 0s-hole state which has [41] spatial symmetry, or (3 1) SU(3) symmetry, for the p^5 part. The amplitude for this component,

which has $L=1$ and $S=0$, is $\sqrt{56/515}$. It remains to transform to a $p^5; J_1 T_1 \otimes (sd); j$ representation using

$$|p^5(\lambda_1 \mu_1) T_1 S_1 \otimes (sd)(2 0) \frac{1}{2} \frac{1}{2}; (1 3) LSJT\rangle = \sum_{L_1 J_1 l j} \langle (\lambda_1 \mu_1) L_1 (2 0) l || (1 3) L \rangle \begin{pmatrix} L_1 & S_1 & J_1 \\ l & \frac{1}{2} & j \\ L & S & J \end{pmatrix} |p^5(L_1 S_1) J_1 T_1 \otimes (sd)(l \frac{1}{2}) j\rangle. \quad (22)$$

Making an assumption of pure SU(3) LS wave functions for the ${}^9\text{Be}$ states in Table 2, we obtain 131 keV for the neutron width to these states from the s_Λ^2 admixture. When the full p -shell wave functions for all energetically available states are used, the calculated neutron width increases to 155 keV ($\Gamma_n(s) = 61$ keV, $\Gamma_n(d) = 93$ keV) and a proton width of 19 keV is obtained for a total nucleonic decay width of 174 keV.

The escape width for the $1^- 0s$ -hole state of ${}^{10}\text{Be}$, calculated using the decay energies appropriate to Ξ^- hypernuclear decay, is 7.4 MeV ($\Gamma_n = 6.6$ MeV). In the purely nuclear case of a proton-hole state in ${}^{11}\text{B}$, the maximum neutron energy is estimated to be 15.2 MeV compared with 22.3 MeV for the hypernuclear case and the calculated escape width will be correspondingly less; the neutron width for the 2^- state, which would be needed in any comparison with data and which enters into the calculation of widths for $0p_\Xi$ states, is about a factor of three smaller than that for the 1^- state because the amplitude of the $p^5[41](sd)$ component in the wave function, which gives access to the low-lying final states of ${}^9\text{Be}$, is much smaller.

3.2.3 Discussion of $0s \Xi^-$ Widths

The partial decay widths that we calculate for the $1^-, {}^{11}\text{B}(g.s.) \otimes 0s_\Xi$ state are collected in Table 3 and compared with the corresponding results of Ikeda *et al.* [22]. Ikeda *et al.* define partial conversion widths Γ_{bb} , Γ_{bc} and Γ_{cc} , where b and c refer to bound and continuum Λ states, and present the contributions from each set of N and Λ orbits for the $0s$ and $0p \Xi^-$ states. Their expression for Γ_{bb} is

$$\Gamma_{bb} = \sum_L \frac{(2L+1)}{4(2l_\Xi+1)} \mathcal{P}(n_N l_N) \frac{\langle n_\Xi l_\Xi n_N l_N | v_{\Xi N, \Lambda\Lambda} | n_{\Lambda_1} l_{\Lambda_1} n_{\Lambda_2} l_{\Lambda_2} \rangle_L^2}{(\epsilon_{n_\Xi l_\Xi} + \epsilon_{n_N l_N} - \epsilon_{n_{\Lambda_1} l_{\Lambda_1}} - \epsilon_{n_{\Lambda_2} l_{\Lambda_2}} + \Delta)^2 + (\Gamma_{n_N l_N}^{(h)})^2/4} \quad (23)$$

where Δ is the $\Xi^- p - \Lambda\Lambda$ mass difference, $\Gamma_{n_N l_N}^{(h)}$ denotes the width of a hole state, and $\mathcal{P}(n_N l_N)$ is a nucleon occupation probability taking into account the fact that the core is not an LS closed shell. Similar expressions for Γ_{bc} and Γ_{cc} involve integrals over the momenta of the emitted Λ particles.

We note that the result $\Gamma_{bb}=551$ keV in Table 3 is arrived at by endowing the nuclear $0s$ -hole state with a width of 10 MeV, taken from the observed width in ${}^{12}\text{C}$. The two-body matrix element, equal to $\sqrt{2}I_0$ for $N\Xi$ states with good isospin, has the value 3.45 MeV and the difference of single-particle energies in the denominator comes

to 5.39 MeV. Our expression can be written in a form similar to that of Eq. 23 with the width term removed from the Breit-Wigner denominator, the full $B_{\Lambda\Lambda}$ of Eq. 3 in place of the Λ single-particle energies and the escape width for the 1^- $0s$ -hole state of ^{10}Be , calculated using the decay energies appropriate to Ξ^- hypernuclear decay, in the numerator. The neutron escape width is calculated to be 6.6 MeV ($\Gamma_n + \Gamma_p = 7.4$ MeV) which, when multiplied by the admixed intensity of 0.02366, gives the neutron width of 155 keV listed in Table 3. The main difference between the two results lies in the additional 4.5 MeV from $\Delta B_{\Lambda\Lambda}$ in the energy denominator. The remaining difference is due to the center of mass factor (0.79), the difference in widths for the $0s$ -hole state and a small difference in two-body matrix elements.

Our p -wave Λ widths are in modest agreement with those of Ikeda *et al.* In our treatment, the $0s$ Λ has a parentage to escape leaving a Λ hypernucleus with the Λ in a $0p$ state. We label this channel by $p_N^{-1}p_\Lambda^b s_\Lambda^c$ in Table 3 and the continuum channels treated by Ikeda *et al.* by $p_N^{-1}p_\Lambda^b(1s0d)_\Lambda^c$. For the latter channels, we give no entry for our calculation since we do not wish to treat the unbound $(sd)_\Lambda$ orbits using harmonic oscillator wave functions. In this approximation, we would have the additional difficulty that the energies of some of these states would be essentially degenerate with the Ξ^- state. Finally, when we treat the nuclear $0s$ -hole state as a discrete state, Λ decay channels are closed on account of the high excitation energy of the residual Λ hypernucleus.

By far the biggest difference between the two calculations arises for p -wave neutron decay. Our relatively large partial width for this channel can be understood in comparison to the calculated width for p -wave Λ emission. Both widths arise from $s_\Lambda p_\Lambda$ admixtures in the Ξ^- state and roughly two of the four p -shell neutrons are linked to low-lying states of ^9Be in the parentage decomposition of the ^{10}Be core states. In comparison, there is only a single p -shell Λ to be emitted. For this rough 2:1 ratio in spectroscopic strength for neutron versus Λ decay, a very good account of the relative Γ_n and Γ_Λ values in Table 3 is obtained once the differences in Γ_{sp} from Tables 1 and 2 for the somewhat different decay energies are taken into account. On the other hand, Ikeda *et al.* endow $0p$ -hole states with a width which is based on the distribution of pickup strength from ^{11}B and use Eq. 23. This does not seem correct since the ^{10}Be core states of interest are discrete states, which can be treated as such in bound-state shell-model calculations, and their spacings have nothing to do with decay widths from the Ξ state beyond their influence in the energy denominators for perturbative estimates of $s_\Lambda p_\Lambda$ admixtures. In fact, in the limit in which the entire $0p$ -hole strength is concentrated in one state of the residual nucleus, namely when $\Gamma_{op}^{(h)} \rightarrow 0$, expression (23) leads to the unacceptable result of a zero width.

3.2.4 Widths of $0p$ Ξ^- states

The 2^+ states that are expected to be most strongly formed in the $^{12}\text{C}(K^-, K^+)$ reaction have a $p_{3/2}$ or $p_{1/2}$ Ξ^- coupled to the ^{11}B ground state. In analogy to $^{12}_\Lambda\text{C}$, the two configurations are expected to mix to form two 2^+ states with a separation of the order of an MeV and a ratio of cross sections which depends on just how strongly the basis configurations, which have equal strength, are mixed.

In shell-model terms, the $0p \Xi$ states are $2\hbar\omega$ states with respect to the $\Lambda\Lambda$ hypernuclear ground state. This means that the $0p \Xi$ state can mix with the $0\hbar\omega$ s_Λ^2 states or with $2\hbar\omega$ $\Lambda\Lambda$ states. In the $0\hbar\omega$ case, the energy denominators are very large, e.g. 41 MeV for $^{10}\text{Be}(gs) \otimes s_\Lambda^2$, and the admixtures will be small. In the $2\hbar\omega$ case, there are many different types of configurations according to the way in which quanta are distributed amongst the nuclear and Λ orbitals. All the different types of configurations are listed in Table 4, together with the kinds of two-body matrix elements which admix them into the $p^7 p_\Xi$ configuration of interest. Expressions for the two-body matrix elements are given in Table 5.

Many features of the decay scheme for the p_Ξ states will be similar to that for the s_Ξ state since the increased excitation energy, $\epsilon_p - \epsilon_s$ for the Ξ , is balanced, for the most important admixed configuration ($s^4 p^6 p_\Lambda^2$), by having an extra p_Λ in the admixed state and the final A-1 system. Given the rather similar spacings of the s_Ξ, p_Ξ and s_Λ, p_Λ orbits, a very crude estimate would give

$$\Gamma_{p_\Lambda}(p_\Xi) \sim \mathcal{R}^2 2\Gamma_{p_\Lambda}(s_\Xi) \quad (24)$$

$$\Gamma_{p_N}(p_\Xi) \sim \mathcal{R}^2 \Gamma_{p_N}(s_\Xi), \quad (25)$$

where \mathcal{R} is the ratio of mixing matrix elements induced by the $\langle p_N p_\Xi | V | p_\Lambda^2 \rangle$ and $\langle p_N s_\Xi | V | s_\Lambda p_\Lambda \rangle$ interactions. The factor of 2 in Eq. 24 appears because either p_Λ from the p_Λ^2 configuration can escape.

The essential features of Eqs. 24 and 25 are apparent from an examination of our calculated partial widths, which are presented in Table 6. Results are presented for both $0p \Xi^-$ basis states, namely a $p_{3/2}$ or a $p_{1/2}$ cascade coupled to the ^{11}B ground state. If the two basis states are allowed to mix, the summed partial widths for the $p_{3/2}$ and $p_{1/2}$ states will be shared by the two resultant 2^+ states. The differences in the partial widths for the two basis states can be understood in terms of the values for the two-body matrix elements in Table 5 and some peculiarities of the recoupling coefficient analogous to the one in Eq. 16. First consider the Γ_Λ widths which result from admixing $p_N^{-1} s_\Lambda (sd)_\Lambda$ configurations, bearing in mind our reservation about treating the $(sd)_\Lambda$ orbits in a bound-state approximation. For the $1s_\Lambda$ orbit, $L_{\Lambda\Lambda}=0$ and the mixing matrix element is very small (Table 5). For the d_Λ orbit, $L_{\Lambda\Lambda}=2$ and the mixing matrix element is substantial. However, the recoupling coefficient vanishes for a $p_{3/2} \Xi$ when the core is in a 2^+ state but is large for a $p_{1/2} \Xi$. The same remarks apply for the $p_N^{-1} p_\Lambda^2$ configuration except that now there is a large two-body matrix element for $L_{\Lambda\Lambda}=0$ which acts to enhance Γ_Λ and Γ_n for a $p_{3/2} \Xi$.

The total decay widths obtained for the $p_{3/2}$ and $p_{1/2}$ cascade states are of the order 1.8 MeV and 1.6 MeV, respectively, which are somewhat larger than the width of 1.2 MeV obtained for the $0s \Xi$ state. These widths should be increased slightly to take into account the effect of unbound Λ orbits that we do not consider in our essentially bound-state approach. The total width associated with the pair of 2^+ states will be larger than that for either of the individual states.

Once again, Ikeda *et al.* obtain very small neutron widths. Also, it is puzzling that they obtain such a small Λ width from the $p_N^{-1} p_\Lambda^b p_\Lambda^c$ configuration given the substantial width that they obtain for the $0s \Xi$ state from the $p_N^{-1} s_\Lambda^b p_\Lambda^c$ configuration.

The end result is that they obtain a width for the $0p\ \Xi$ state which is considerably smaller than that for the $0s\ \Xi$ state. In addition, it is not clear which state their calculated $0p_{\Xi}$ width applies to.

4 Summary

The purpose of the present paper was to obtain an estimate for the widths of $0s$ and $0p\ \Xi^{-}$ states which can be produced in the $^{12}\text{C}(K^{-}, K^{+})$ reaction. We used standard shell-model techniques to make estimates of the mixing of Ξ and $\Lambda\Lambda$ hypernuclear states and hence obtain estimates for the neutron and Λ decay widths of the Ξ hypernuclear states. For a $\Xi^{-}p \rightarrow \Lambda\Lambda$ interaction based on G matrices calculated using a version of the Nijmegen Model D interaction for the baryon-baryon interaction in the strangeness -2 sector, decay widths in the range 1 – 2 MeV were obtained for these states.

A large part of the decay width for the Ξ hypernuclear states is attributed to p -wave emission of both neutrons and Λ particles following the conversion of the Ξ^{-} and a p -shell proton. The nuclear core states on which the admixed $\Lambda\Lambda$ hypernuclear states are based are essentially discrete states which are directly amenable to our straightforward shell-model treatment. On the other hand, the nuclear $0s$ -hole strength, which is relevant when the Ξ^{-} converts on an s -shell proton, appears as a broad resonance-like distribution in knockout reactions. In realistic shell-model calculations, the $0s$ hole strength is spread over a considerable range of excitation energy with each fragment having a substantial decay width into nucleon, and perhaps other, channels. For our estimates of Ξ hypernuclear decay widths, we have concentrated the $0s$ -hole strength in a single state related to the hypernuclear core state and have, in effect, calculated the neutron decay width of this state for the energetics determined by the energy of the Ξ hypernuclear state. This should give quite a reasonable estimate of the escape width for the Ξ state. If the Ξ state comes close in energy to $\Lambda\Lambda$ states based on major fragments of the $0s$ -hole strength, it could be fragmented by configuration mixing and thus acquire a spreading width. This point needs to be investigated in more detailed shell-model calculations. However, the separation of about 10 MeV between the Ξ states of interest for $A=12$ and the centroid of $\Lambda\Lambda$ states based on nuclear $0s$ -hole states suggests that the Ξ states will not be appreciably fragmented by this mechanism. For somewhat heavier systems, this may not be the case as the $0s$ -hole state becomes more deeply bound. For example, we estimate that the 10 MeV separation for $A=12$ is reduced to about 3 MeV for $A=16$.

A caveat to the results obtained in this paper is that the decay widths of Ξ hypernuclear states depend quadratically on the strength of the effective $\Xi N - \Lambda\Lambda$ interaction in the medium, about which there is, at present, considerable uncertainty. If the strength is similar to that deduced from G-matrix calculations [18] using the Nijmegen Model D interaction, there should exist Ξ hypernuclear states narrow enough for study using the (K^{-}, K^{+}) reaction with a resolution of ~ 2 MeV. Of course, it must also be remembered that we have assumed the existence of a Ξ -nucleus potential deep enough to bind Ξ single-particle states. For in-flight studies of the (K^{-}, K^{+})

reaction, the high-quality and intensity of K^- beams from the D6 beamline [21] at the Brookhaven AGS could provide a promising means to search for Ξ hypernuclear states. However, it is evident from the discussion in Sec. 2 that such studies are only on the borderline of feasibility and an improved spectrometer with high efficiency and good resolution will be required. As emphasised in the first two sections of this paper, the physics to be gained from a successful observation of Ξ hypernuclear states is considerable.

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Tables

Table 1: $s_\Lambda p_\Lambda$ mixing amplitudes (b) and p -wave partial Λ decay widths (Γ_Λ) for the $1^- 0s \Xi^-$ state of $^{12}_{\Xi}\text{Be}$. Γ_{sp} for a decay energy E_Λ is calculated for an equivalent square well of radius $R = 3.33$ fm, depth $V_0 = 25$ MeV and reflection factor $f = 2.5$ (see Eq. 12). All energies are in MeV. The total admixed intensity is 0.88% and the total p -wave Λ decay width is 398 keV.

Core State	ΔE_p	$b(\alpha_n J_n^\pi)$	E_Λ	Γ_{sp}	Γ_Λ
0_1^+	21.5	-0.0422	16.8	56.4	0.100
2_1^+	18.1	-0.0440	13.4	48.7	0.094
2_2^+	15.5	0.0654	10.8	41.9	0.179
2_3^+	12.1	-0.0275	7.4	31.0	0.024

Table 2: Neutron spectroscopic factors (S) and p -wave partial neutron decay widths (Γ_n) for the $1^- 0s \Xi^-$ state of $^{12}_{\Xi}\text{Be}$. Γ_{sp} for a decay energy E_n is calculated for an equivalent square well of radius $R = 3.33$ fm, depth $V_0 = 40$ MeV and reflection factor $f = 2.5$ (see Eq. 12). All energies are in MeV. The total p -wave neutron decay width is 622 keV.

^9Be State	$E_x(^9\text{Be})$	S	E_n	Γ_{sp}	Γ_n
$\frac{3}{2}^-_1$	0.0	0.00617	12.0	43.6	0.269
$\frac{5}{2}^-_1$	2.4	0.00054	9.6	36.9	0.020
$\frac{1}{2}^-_{21}$	2.8	0.00168	9.2	35.7	0.060
$\frac{3}{2}^-_{22}$	4.7	0.00047	7.3	29.5	0.014
$\frac{7}{2}^-_{21}$	6.4	0.00806	5.6	23.2	0.187
$\frac{5}{2}^-_{22}$	7.4	0.00379	4.6	19.0	0.072

Table 3: Summary of partial contributions to the width of a $J^\pi=1^-, 0s_\Xi$ state in $^{12}_{\Xi^-}\text{Be}$ in the calculations of Ikeda *et al.* [22] and this work. The different contributions are labelled in the manner of Ref. [22] by $l_N^{-1}l_\Lambda^i l_\Lambda^j$ with $i,j=b,c$ where b and c mean a Λ particle in bound and continuum states, respectively. We compare the Γ_{bb} and Γ_{bc} of Ref. [22] with our values for $\Gamma_n \simeq \Gamma_N$ and Γ_Λ . In addition, we give the maximum decay energies E_n and E_Λ , using our kinematics, for each partial decay channel. With our use of a discrete energy for the nuclear $0s$ -hole state, some channels are closed. An asterisk indicates channels with unbound Λ orbits that we do not treat.

Channel	Γ_{bb} keV	E_n MeV	Γ_n keV	Channel	Γ_{bc} keV	E_Λ MeV	Γ_Λ keV
$s_N^{-1}s_\Lambda^b s_\Lambda^b$	551	22.3	155	$s_N^{-1}s_\Lambda^b s_\Lambda^c$	111	<0	
$s_N^{-1}p_\Lambda^b p_\Lambda^b$	17	1.7		$s_N^{-1}p_\Lambda^b p_\Lambda^c$	30	<0	
$p_N^{-1}s_\Lambda^b p_\Lambda^b$	25	12.0	622	$p_N^{-1}s_\Lambda^b p_\Lambda^c$	281	16.8	398
				$p_N^{-1}p_\Lambda^b s_\Lambda^c$		6.5	158
				$p_N^{-1}p_\Lambda^b 1s_\Lambda^c$	23	6.5	*
				$p_N^{-1}p_\Lambda^b d_\Lambda^c$	118	6.5	*

Table 4: Configurations and two-body mixing matrix elements which enter into a shell-model calculation for studies of the $0p \Xi^-$ states.

Configurations	Two-body matrix elements
$s^4 p^7 p_\Xi$	
$(\alpha s^3 p^8 + \beta s^4 p^6(sd))s_\Xi$	$\langle s_N p_\Xi V p_N s_\Xi \rangle, \langle p_N p_\Xi V (sd)_N s_\Xi \rangle$
$s^4 p^6 p_\Lambda^2$	$\langle p_N p_\Xi V p_\Lambda^2 \rangle$
$s^4 p^6 s_\Lambda(sd)_\Lambda$	$\langle p_N p_\Xi V s_\Lambda(sd)_\Lambda \rangle$
$(\alpha' s^3 p^7 + \beta' s^4 p^5(sd))s_\Lambda p_\Lambda$	$\langle s_N p_\Xi V s_\Lambda p_\Lambda \rangle$
$(\alpha'' s^4 p^4(sd)^2 + \beta'' s^3 p^6(sd) + \gamma'' s^4 p^5(pf))s_\Lambda^2$	0
$s^4 p^6 s_\Lambda^2$	$\langle p_N p_\Xi V s_\Lambda^2 \rangle$

Table 5: Expressions in terms of Talmi integrals for two-body mixing matrix elements which enter into a shell-model calculation for studies of the $0p \Xi^-$ states. The $N\Xi$ configurations are proton- Ξ^- in nature. They and the $\Lambda\Lambda$ configurations have $S=0$ and the L specified in column 2. The values of the two-body matrix elements for the interaction that we use are given in the final column.

Two-body matrix elements	L	Expression	Value (MeV)
$\langle p_N p_\Xi V p_\Lambda^2 \rangle$	0	$\frac{1}{4}(5I_0 - 6I_1 + 5I_2)$	2.607
$\langle p_N p_\Xi V p_\Lambda^2 \rangle$	2	$\frac{1}{2}(I_0 + I_2)$	1.215
$\langle p_N p_\Xi V 0s_\Lambda 1s_\Lambda \rangle$	0	$-\frac{1}{4}(I_0 - 6I_1 + 5I_2)$	-0.212
$\langle p_N p_\Xi V 0s_\Lambda 0d_\Lambda \rangle$	2	$\frac{1}{2}(I_0 - I_2)$	1.180
$\langle s_N p_\Xi V s_\Lambda p_\Lambda \rangle$	1	$\sqrt{\frac{1}{2}}I_0$	1.693
$\langle p_N p_\Xi V s_\Lambda^2 \rangle$	0	$-\frac{\sqrt{3}}{2}(I_0 - I_1)$	-1.826

Table 6: Summary of partial contributions to the width of a $J^\pi=2^+$, $0p_\Xi$ states in $^{12}_{\Xi^-}\text{Be}$ in the calculations of Ikeda *et al.* [22] and this work, which includes results for both $0p_{3/2}$ and $0p_{1/2}$ Ξ^- particles coupled to the ^{11}B ground state. See the caption to Table 3 for notation.

Channel	Γ_{bb} keV	E_n MeV	$\Gamma_n(p_{3/2})$ keV	$\Gamma_n(p_{1/2})$ keV	Channel	Γ_{bc} keV	E_Λ MeV	$\Gamma_\Lambda(p_{3/2})$ keV	$\Gamma_\Lambda(p_{1/2})$ keV
$s_N^{-1}s_\Lambda^b p_\Lambda^b$	104	21.2	64	84	$s_N^{-1}s_\Lambda^b p_\Lambda^c$	115	4.0	268	268
$p_N^{-1}s_\Lambda^b s_\Lambda^b$	1	31.5	89	11	$s_N^{-1}p_\Lambda^b 1s_\Lambda^c$	1	<0		
$p_N^{-1}p_\Lambda^b p_\Lambda^b$	7	10.9	368	267	$s_N^{-1}p_\Lambda^b d_\Lambda^c$	8	<0		
$p_N^{-1}s_\Lambda^b s_\Lambda^b d_\Lambda^b$		10.9	97	205	$p_N^{-1}s_\Lambda^b s_\Lambda^c$		26.0	89	13
					$p_N^{-1}s_\Lambda^b 1s_\Lambda^c$	1	26.0	3	1
					$p_N^{-1}s_\Lambda^b d_\Lambda^c$	69	26.0	85	284
					$p_N^{-1}p_\Lambda^b p_\Lambda^c$		15.7	738	508
					$p_N^{-1}p_\Lambda^b 1p_\Lambda^c$	57	15.7	*	*
					$p_N^{-1}p_\Lambda^b f_\Lambda^c$	17	15.7	*	*

Figure captions

Fig. 1: Energy spectrum and decay thresholds of Ξ and $\Lambda\Lambda$ hypernuclear configurations for the case of a ^{12}C target. The cross hatched area indicates that the nuclear $0s$ -hole strength is fragmented. The 0^+ and 2^+ designations at the right of the figure refer to states of the ^{10}Be core in $\Lambda\Lambda$ hypernuclear states. The main neutron and Λ decay modes of the $0s$ Ξ^- -hypernuclear state are indicated in the center of the figure.

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